We look forward to extending the range of reusable seals to more rigorous vacuum applications as new seal materials are discovered and perfected.

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# Design Guides for Tapered Transition Sections for Pressure Vessels

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**Tapered transition sections to generate constant strength (and thus minimum weight) design criteria for symmetrically loaded thin shells of revolution are analyzed. A finite difference solution (digital computer) to Love's first approximate thin-shell theory is used. Tapered transition sections are considered for a cylindrical pressure vessel ($R/t$ = 100) closed by either hemispherical, torispherical, or 2:1 ellipsoidal heads.**

### Introduction

At the juncture of two dissimilar shells, shear and bending moments arise which raise the stress level above the desired membrane stress. A correctly designed transition section will minimize the effects of these discontinuity forces and moments, and a shell structure will result that has nearly constant strength. If the design or membrane stress level is close to the yield point of the material, then a minimum weight design will result. The present study determines the dimensions of tapered transition sections at the junctures of various heads with the main cylindrical wall of a minimum weight pressure vessel.

The normal method[1] of solving the juncture problem is to generate the "edge influence coefficients" of each of the shells and solve the equilibrium and compatibility equations for the forces and moments at the juncture. For instance, the variation of the meridional bending moment in a spherical cap near a boundary is

$$M_\phi = -t(C_2 \cos\beta + C_1 \sin\beta)e^{-\beta} \qquad (1)$$

where $t$ is the shell thickness, $C_1$ and $C_2$ are constant coefficients related to edge loads and shell geometry, and $\beta = S/(2R_\theta t)^{1/2}$, where $R_\theta$ is the radius of curvature in the circumferential direction, and $S$ the surface length from boundary.

Evaluation of Eq. (1) indicates that the applied edge moment is reduced to less than 10% of its original value when $\beta \sim 2$, or $S \sim 3(R_\theta t)^{1/2}$. The so-called characteristic length $(R_\theta t)^{1/2}$ is extremely important in the design of tapered transition sections because the membrane stress condition will predominate in little more than three characteristic lengths from a boundary. In a spherical cap, this fixes the maximum length of the transition section.

By means of a digital computer solution[2] based on the linear theory of symmetrically loaded thin shells of revolution (Love's first approximation with small displacements), one can calculate the edge influence coefficients and solve the juncture problem. The computer analysis will generate directly the forces, moments, displacements, and rotations at

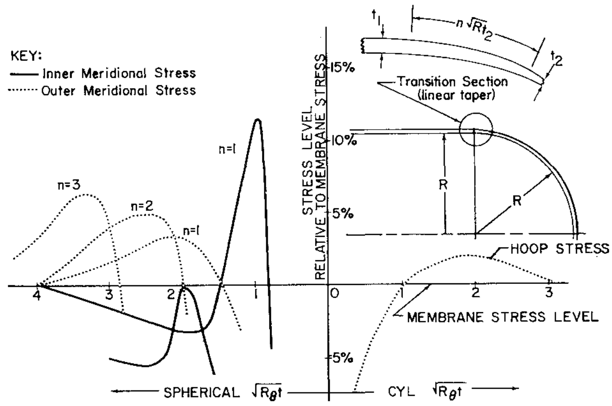


**Fig. 1 Stresses at the juncture of a hemispherical head and cylinder with linear taper in transition section.**

the juncture. In this note, tapered transition sections are examined for three kinds of shell junctures (Figs. 1–3), and several tapers are considered for each. In all cases, the cylindrical section has a radius/thickness ratio of 100. The heads are selected on a rough basis of equal weight, with radius-of-curvature/thickness ratios of the order 200. These dimensions fall within the range that may be analyzed by linear elastic thin-shell theory (i.e., $500 > R/t > 50$).

### Hemispherical Head-Cylinder Junctures (Fig. 1)

Constant strength design would dictate that the thickness of the head be one-half the thickness of the cylinder. Discontinuity stresses will arise at the joint due to the differences in bending resistance and radial stiffness of the shell edges when the vessel is pressurized; hence bending moments and shear forces will be generated. For an abrupt joint (no taper), the maximum hoop stress in the head will be 7% above the membrane stress.

Figure 1 shows the maximum stresses in the head and cylinder with linearly tapered sections of length $n(R_\theta t)^{1/2}$, where $n$ = 1, 2, and 3. For $n$ = 1 (a linear taper which doubles the thickness of the shell in one characteristic length from the joint), the meridional stress on the inner surface is 11% higher than the membrane stress. As $n$ is increased, this stress peak is reduced until the meridional stress on the outside surface begins to increase. The maximum stress in each taper occurs near or at the base of the taper in the head. A minimum stress, approximately 5% above membrane stress, is noted when $n$ is between 1.5 and 2.

A taper was examined that was continuous in the first derivative. In that taper, the stress was further reduced to 4% above the membrane stress level. Thus for a cylindrical pressure vessel with hemispherical ends, the working stress level may be raised 3% by the use of tapered transition sections. This will effect a possible 3% reduction in weight.

### Torispherical Head—Cylinder Junctures (Fig. 2)

Some torispherical head designs[3] appear to have slight weight advantages over the full hemispherical head. For the constant thickness, torispherical head with membrane-section stress equal to that of the cylindrical shell, the head thickness is 52.2% of the shell thickness. The torus-sphere juncture is 3 characteristic lengths away from the torus-cylinder juncture. Tapered transitions within the toroidal section (curves 1–3) lead to stresses 30–40% above membrane level. To reduce the stresses in this area, the full cylindrical thickness can be maintained through the toroidal sections, so that the tapered transition occurs in the spherical sector [curves $E$-1, $E$-2, and $E$-3, where the number signifies the value of $n$ for the length of the linear taper, $n(R_\theta t)^{1/2}$]. The $E$-3 transition reduces the stress level to within 6% of the membrane stress level. (There is little to be gained in reducing the stress level further, because the maximum hoop stress in the cylindrical

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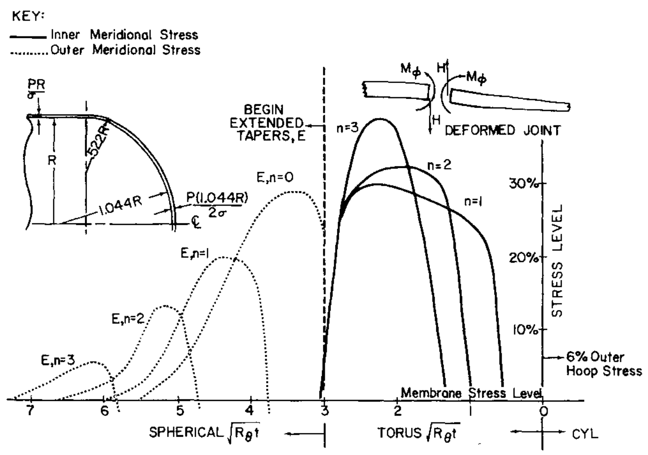


**Fig. 2 Stresses in a torispherical head with tapered transition sections.**

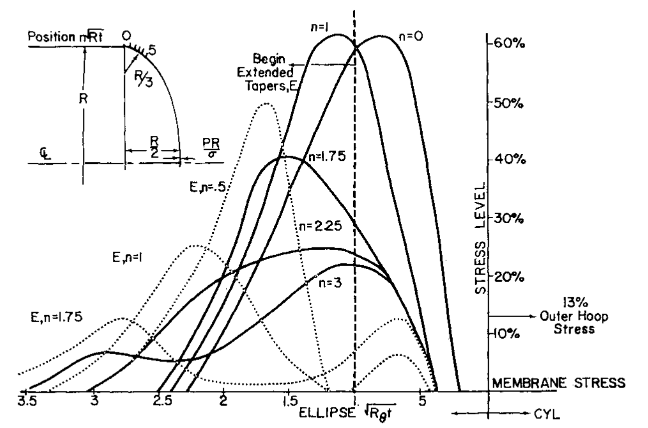


**Fig. 3 Stresses in a 2:1 ellispoidal head with tapered transition sections.**

section has a 6% stress peak at about 2 characteristic lengths from the torus-cylinder joint). For the latter transitions, stresses below membrane level were calculated in the toroidal section; this indicates that a double taper would reduce the transition section weight (a good subject for further study).

### Ellipsoidal Head—Cylinder Juncture (Fig. 3)

The discontinuity stresses at the juncture of a 2:1 ellipse and a cylinder are high, because the ellipse has an equal but compressive membrane stress in the hoop direction. The stresses are particularly high in the meridional direction in the "knuckle" of the ellipse. The head studied here has a constant thickness and a weight equal to that of the hemispherical head; its thickness is 65% of that of the cylinder. (An excellent analysis of ellipsoidal heads has been made in Ref. 4, which substantiates the stress levels noted here.)

Maximum stresses for linear tapers with $n = 0$, 1.75, 2.25, and 3 are shown in Fig. 3. For $n \leq 1$, maximum stress level is 60% above membrane stress level; for $n = 2.25$ to 3, it has fallen to about 22% of the cylindrical membrane stress level. The maximum allowable $n$ is probably near 3, because this would give a tapered length equal to 20% of the head surface length for a shell of $R/t = 100$. Marked reductions in stress are noted when "extended" tapers ($E$, $n$ curves) are used. (The full thickness of the cylinder is carried 1 characteristic length into the knuckle of the ellipse, then the wall section is thinned to the required 65% of the cylinder thickness.) Maximum stresses now occur on the exterior wall rather than the interior, caused by a change in bending moments. These bending stresses are reduced when the taper covers 1.75 characteristic lengths. (Longer tapers were not used, because bending stresses increased at the juncture, which caused an increase in the hoop stresses in the cylinder.) The maximum stress levels in the cylinder and the ellipse for this "$E$, $n = 1.75$" taper are approximately equal—about 12% higher than the maximum cylindrical membrane stress.

### Conclusions

Design guides have been developed for tapered transition sections for thin-shell ($500 > R/t > 50$), minimum-weight pressure vessels. For the hemispherical head-cylinder joint, a linearly tapered transition $2(R_\theta t)^{1/2}$ in length on the head side will reduce the discontinuity stress level to within 5% of the membrane stress; the maximum stress level will occur at the thin end of the taper; and, therefore, all welds should be removed from that immediate area. For maximum stress reduction, the taper should be faired into the head.

For the torispherical shell of Fig. 2, the taper should be placed on the spherical side of the joint over two characteristic lengths. The resulting meridional stress level in the head will be within 6% of the cylinder membrane stress level. A hoop stress peak of the same magnitude will be apparent in the cylinder at a position 2 characteristic lengths for the joint.

For the 2:1 ellipsoidal head of constant thickness, the taper should begin one characteristic length in from the joint and should cover 1.5 to 2 characteristic lengths of the ellipsoidal surface.

In summation, minimum-weight pressure vessel design requires that bending stresses be reduced wherever possible. This can be done by the use of tapered transition sections at the juncture of dissimilar shells. However, the design of these transition sections is related closely to the geometry of the head except in the case of the full hemisphere. Therefore, in nonhemispherical closures, caution should be exercised when the transition section is designed.

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# Solar Simulation in Space Environment

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At present solar simulation is in its infancy. Evolution of the art has not yet reached the point where the trend toward standardization of design has manifested itself. Many current solar simulator systems represent compromises from the ideal. In the following paragraphs some of the more important design factors being used and the types of errors

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